

MATHS - 2003 (Outside Delhi)

SECTION A

Question numbers 1 to 10 carry 3 marks each.

Q. 1. Prove that : $(1 - \sin A)(\sec A + \tan A) = 1$

Q. 2. Find the GCD and LCM of the following polynomials $24(x^3 + 9x^2 + 20x), 28(x^4 + x^3 - 12x^2)$

Ans. GCD = $(4x^2x + 4)$; LCM = $168x^2(x - 3)(x + 4)(x + 5)$

Or

If the GCD of $x^2 - ax - 6$ and $x^2 - 2x + b$ is $(x - 6)$ find the values of a and b .

Ans. (a = 5; b = -24)

Q. 3. Find the value of k for which the following system of linear equations has infinite number of solutions : $x + (k + 1)y = 5, (k + 1)x + 9y = 8k - 1$

Ans. (K = 2)

Q. 4. Express the following as a rational expression in lowest form:

$$\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}; x \neq \pm 3, x \neq 4$$

Ans. $\frac{2(x + 4)}{(x + 3)}$

Q. 5. Eleven balls are serially numbered and placed in an urn. Find the probability that a ball drawn will be: (a) odd numbered ball (b) even numbered ball (c) prime numbered ball.

Ans. (a) $\frac{6}{11}$; (b) $\frac{5}{11}$; (c) $\frac{5}{11}$

Q. 6. If the mean of the following frequency distribution is 8, find the value of p.

x	3	5	7	9	11	13
y	6	8	15	p	8	4

Ans. (p = 3)

Or

In a class of 25 students, 15 are boys. The mean weight of boys is 50kg and that of girls is 45 kg. Find the mean weight of the class.

Ans. (48 kg.)

Q. 7. Find the co-ordinates of the points which divide internally the line joining (1, -3) and (-3, 9) in the ratio 1 : 3.

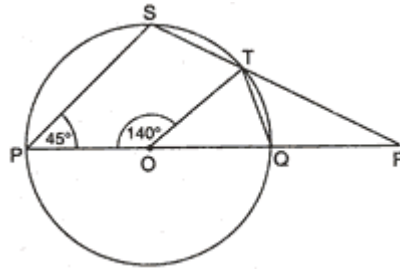
Ans. (0, 0)

Q. 8. The 4th term of an A.P. is equal to 3 times the first term and the 7th term exceeds twice the 3rd term by 1. Find the 1st term and common difference.

Ans. (3 : 2)

Q. 9. D and E are points on the sides AB and AC respectively of $\triangle ABC$ such that DE is parallel to BC, and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of $\triangle DEF$ and $\triangle BCF$.
Ans. (16/81)

Q. 10. From Figure 1, find $\angle RTQ$ and $\angle RQT$.
Ans. ($\angle RQT = 110^\circ$; $\angle RTQ = 45^\circ$)



SECTION B

Question numbers 11 to 20 carry 4 marks each.

Q. 11. Solve the following system of linear equations graphically:
 $3x - 5y = 19$, $3y - 7x + 1 = 0$

Does the point (4, 9) lie on any of the lines? Write its equation.

Ans. [$x = -2$, $y = -5$, $3x - 7y + 1 = 0$]

Q. 12. Solve the following system of linear equations:

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2, (a + b)(x + y) = a^2 + b^2$$

Ans. $\left(x = a + b, y = \frac{-2ab}{a + b} \right)$

Or

The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction becomes. Find the fraction.

Ans. (3/5)

Q. 13. Determine value(s) of p for which the quadratic equation $x^2 + px + 1 = 0$ has real roots.

Ans. $x^2 + px + 1 = 0$

Or

*Determine the 2nd term and the r th term of an A.P., whose 6th term is 12 and 8th term is 22.

Ans. (5r - 18)

Q. 14. The total annual income of Rajesh (excluding HRA) is Rs.1,83,000. He pays Rs.3,500 semi annually towards life insurance premium. How much should he contribute per month towards Provident Fund to get maximum rebate. Also find his tax liability for the financial year.

Assume the following for calculating income tax:

(a) Standard Deduction :

(i) 40% of the total income subject to a maximum of Rs.30,000 in

case the total annual income is up to Rs.1,00,000
(ii) Rs.30,000 in case the total annual income is from Rs.1,00,001 to Rs.5,00,000

(b) Rates of income tax :

Slab	Income tax
(i) Upto Rs.50,000	No tax
(ii) From Rs.50,001 to Rs. 60,000	10% of the amount exceeding Rs. 50,000
(iii) From Rs.60,001 to Rs. 1,50,000	Rs.1,000+20% of the amount exceeding Rs. 60,000
(iv) Above Rs.1,50,000	Rs.19,000+30% of the amount exceeding Rs.1,50,000
(c) Rebate in tax	(i) 20% of the amount of saving Subject to maximum Rs. 14,000, if gross income is upto Rs. 1,50,000 (ii) 15% of the amount of saving subject to a maximum of Rs.10,500, if gross income is above Rs. 1,50,000 but not exceeding Rs. 5,00,000

Q. 15. A housing society charges for a flat Rs. 16,00,000 cash or Rs. 5,85,500 cash down payment and three equally half yearly instalments. If the society charges 16% per annum interest compounded half-yearly, calculate the value of each instalment. Find also the total interest charged.

Ans. (Rs. 3,93,660; Rs.1,66,480)

Q. 16. In what ratio does the point $\frac{1}{2}, 6$ divide the line segment joining the points $(3, 5)$ and $(-7, 9)$?
Ans. (1 : 3)

Q. 17. Without using trigonometric tables, evaluate the following:

$$2 \frac{\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ + \tan 15^\circ \tan 25^\circ \tan 60^\circ \tan 65^\circ \tan 75^\circ$$

Ans. ($\sqrt{3}$)

Q. 18. In $\triangle ABC$, $\angle A$ is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that $AE \times AC = AD \times AB$.

Q. 19. Construct a $\triangle ABC$ in which BC = 4 cm, $\angle A = 50^\circ$ and altitude through A is 3 cm. Also write the steps of construction.

Q. 20. In the month of January 2003, a householder spent his monthly salary amounting to Rs. 14,400 on different items as given below:

Items	Clothing	Food	House Rent	Education	Misc.
Amount spent (in Rs.)	1,200	8,000	2,400	800	2,000

SECTION C

Question numbers 21 to 25 carry 6 marks each.

Q. 21. The length of the hypotenuse of a rt. angled triangle exceeds the base by 1 cm and also exceeds twice the length of the altitude by 3 cm. Find the length of each side of \triangle .

Ans. (12 cm; 13 cm; 5 cm)

Or

"If twice the area of smaller square is subtracted from the area of a large square, the result is 14 cm^2 However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203 cm^2 Determine the sides of squares.

Ans. (smaller side = 5 cm; larger side = 8cm)

Q. 22. In a triangle, if the square on one side is equal to the sum of the squares on the other two sides, prove that the angle opposite the first side is a right angle.

Use the above theorem and prove the following:

In triangle $\triangle ABC, AD \perp BC$ and $BD = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.

Q. 23. If $APAB$ is a secant to a circle intersecting the circle at A and B and PT is a tangent segment, prove that $PA \times PB = PT^2$

Using the above theorem, prove the following:

Two circles intersect each other at P and Q. From a point R on PQ produced, two tangents RB and RC are drawn to the two circles touching them at B and C. Prove that $RB = RC$.

Q. 24. The internal and external radii of a hollow sphere are 3 cm and 5 cm respectively. The sphere is melted to

form a solid cylinder of height $2\frac{2}{3}$ cm. Find the diameter and curved surface area of the cylinder.

Ans. (14 cm; 117.33 cm^2)

Q. 25. The angles of elevation and depression of the top and bottom of a light-house from the top of a building, 60 m high, are 30° and 60° respectively. Find

(i) the difference between the heights of the light-house and the building.

(ii) distance between the light-house and the building.

Ans. (20 m; 34.64 m)

Or

*The height of a house subtends a right angle at the opposite window. The angle of elevation of the window from the base of the house is 60° . If the width of the road is 6 m, find the height of the house.

Ans. (13.856 m)