

MATHEMATICS—2005 (Delhi)

SECTION A

Question numbers 1 to 10 carry 3 marks each.

Q. 1. Express the following as a rational expression in lowest terms:

$$\frac{x^2 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1} \div \frac{x^2 + 2x + 4}{x^2 + 2x - 3}$$

Ans:
$$\frac{x^2 + 7x + 12}{x - 1}$$

Q. 2. Find 10th term from end of the A.P. 4, 9, 14, ... 254.

Ans: (209)

Q. 3. Solve the following system of linear equations:

$$ax + by = a - b$$

$$bx - ay = a + b$$

Ans: (x = 1, y = -1)

Q. 4. Find the L.C.M. of the following polynomials:

$$2x^3 - 128, x^2 - 9x + 20, x^2 - 16$$

Ans:
$$[2(x - 4)(x + 4)(x - 5)(x^2 + 4x + 16)]$$

Q. 5. Solve for x.

$$\frac{1}{a + b + x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; a \neq 0, b \neq 0, x \neq 0$$

Ans: (c/a, -b/a)

Or

Solve for x.

$$abx^2 + (b^2 - ac)x - bc = 0$$

Ans: (c/a, -b/a)

Q. 6. Find the number of terms of the A.P. 54, 51, 48 ... so that their sum is 513.

Ans:(n = 18 or n = 19)

Or

If the nth term of an A.P. is $(2n + 1)$, find the sum of first n terms of the A.P.

Ans:n (n + 2)

Q. 7. A loan of Rs 10,815 is to be returned in three equal half-yearly instalments.

Calculate the amount of each instalment, if the rate of interest is $13\frac{1}{3}\%$ per annum, compounded half-yearly.

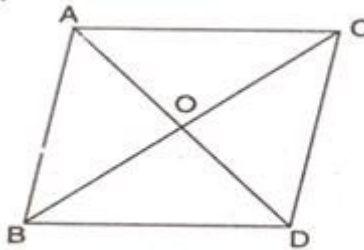
Ans:(Rs. 4,096)

Q. 8. A fan is available for Rs 970 cash or Rs 210 as cash down payment followed by three equal monthly instalments of Rs 260 each. Find the rate of interest charged under instalment plan.

Ans:(16% p.a.)

Q. 9. In the figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. AD and BC intersect

at O. prove that $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$



Q. 10. OD is perpendicular to a chord AB of a circle whose centre is O. If BC is a diameter, prove that $CA = 2 OD$.

SECTIONS B

Question numbers 11 to 20 carry 4 marks each.

Q. 11. Solve the following system of equations graphically :

$$2x - 3y = -4$$

Also find the points where the lines meet the x-axis.

Ans: B(5, 0), C(-2, 0)

Q. 12. The sum of two numbers a and b is 15, and the sum of their reciprocals $1/a$ and $1/b$ is $3/10$. Find the numbers a and b.

Ans: (a = 10, b = 5 or a = 5, b = 10)

Q. 13. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are needed to empty the bowl?

Ans: (54)

Q.14 Prove that:

$$\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \cos^2 B}$$

find the value of:
$$\frac{-\tan \theta \cdot \cot(90^\circ - \theta) + \sec \theta \cdot \operatorname{cosec}(90^\circ - \theta) + \sin^2 35^\circ + \sin^2 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 30^\circ \tan 70^\circ \tan 80^\circ}$$

Ans: $(2\sqrt{3})$

Q. 15. Draw a circle of radius 3.5 cm. From a point P outside the circle at a distance of 6 cm from the centre of circle, draw two tangents to the circle.

Q. 16. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6, -1);

(1, 3) and (x, 8) respectively.

Ans: (x = 5 or x = -3)

Or

Find a point on x-axis which is equidistant from the points (7, 6) and (-3, 4).

Ans: (3, 0)

Q. 17. The line-segment joining the points (3, -4) and (1, 2) is trisected at the points P and Q. If the

Coordinates of P and Q are (p, -2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q.

Ans: (p=7/3 ; q=0)

Q. 18. Find the mean of the following distribution:

Class	Number of Students
4-8	2
8-12	12
12-16	15
16-20	25
20-24	18
24-28	12
28-32	13
32-36	3

Ans:(19-92)

Q. 19. Given below is the expenditure of a person on different items out of his salary of Rs 14,400.

Item:	Clothing	Food	Rent	education	others	G. total
Expn. : (in Rs.)	2,800	3,600	3,600	1,800	2,600	14,400

Draw a pie-chart to depict the above data.

Q. 20. A card is drawn at random from a well shuffled pack of 52 cards. Find the Probability that the card drawn is neither a red card nor a queen.

Ans:(6/13)

SECTION C

Question numbers 21 to 25 carry 6 marks each.

Q. 21. Prove that in a right angled triangle the square on the hypotenuse is equal to the sum of the squares on other two sides.

Using the above result, prove that the sum of squares on the sides of a rhombus is equal to sum of squares on its diagonals.

Q. 22. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and flag pole mounted on it.

Ans: (5.19 m, 10.38m)

Or

From a building 60 metres high the angles of depression of the top and bottom of a lamppost are 30° and 60° respectively. Find the distance between the lamppost and building. Also find the difference of height between building and lamppost.

Ans:(34.6 m, 20m)

Q. 23. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of Rs 80 per sq. m .

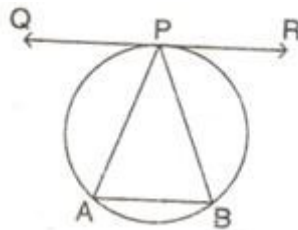
Ans; (Rs 82,720)

Or

The radii of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area.

Ans:(7599-43 cm²)

Q. 24. If a line touches a circle and from the point of contact a chord is drawn, prove that the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.



Using the above theorem, prove the following: P is mid point of arc APB. Prove that tangent QR drawn at P to the circle is parallel to AB.

Q. 25. The monthly income of Ruchi (age 30 years) is Rs. 18,200 (excluding HRA). She donates Rs. 8,400 towards Prime Minister Relief Fund (Relief 100%) and Rs. 12,000 to an orphanage (Relief 50%). She contributes Rs. 4,500 per month towards Provident Fund and Rs. 2,500 per quarter as LIC premium. She purchases National Saving Certificates worth Rs. 8,000. She pays Rs. 900 per month as income-tax for the first 11 months. Calculate her income-tax liability for last month of the year.

For calculating income tax, use the following:

- (a) Standard Deduction : (i) Rs. 30,000 if incomes is up to Rs.5,00,000
(ii)Rs. 20,000 if income is more than Rs.5,00,000

(b) Rates of income tax :

Slab	Income tax
(i) Upto Rs.50,000	No tax
(ii) From Rs.50,001 to Rs. 60,000	10% of the amount exceeding Rs. 50,000
(iii)From Rs.60,001 to Rs. 1,50,000	Rs.1,000+20% of the amount exceeding Rs. 60,000
(iv) Above Rs.1,50,000	Rs.19,000+30% of the amount exceeding Rs.1,50,000
(c) Rebate in tax	(i) 20% of the savings subject to a maximum of Rs.14,000 if the gross incomes is upto Rs.1,50,000. (ii) 15% of the savings subject to a maximum is upto Rs.10,500 if the gross income is between Rs.1,50,001 to Rs.5,00,000.

(d) Educational cess

2% of the income tax payable

(e) Special Rebate

For female, maximum Rs. 5,000 over and above the rebate on savings.

(Ans;Rs. 1,014)