

## SECTION - A

**Q. 1.** If  $A = \begin{pmatrix} 3 & -5 \\ -4 & 2 \end{pmatrix}$ , show that  $A^2 - 5A - 14I = O$ .

**Q. 2.** Using properties of determinants, solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

**Q. 3.** An urn contains 7 white, 5 black and 3 red balls. Two balls drawn at random. Find the probability that

- (i) Both the balls are red
- (ii) One ball is red, the other is black
- (iii) One ball is white

**Q. 4.** A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success.

**Q. 5.** Evaluate:  $\int \sin^4 x \, dx$ .

**Q. 6.** Evaluate:  $\int \frac{\sin^{-1} x}{x^2} \, dx$ .

**Q. 7.** Form the differential equation corresponding to  $y^2 = a(b - x^2)$  where  $a$  and  $b$  are arbitrary constants.

**Q. 8.** Solve the differential equation:  $(1 + e^{2x})dy + e^x(1 + y^2)dx = 0$   
Given the  $y = 1$ , when  $x = 0$

Or

Solve the differential equation:  $x \frac{dy}{dx} - y - 2x^3 = 0$ .

**Q. 9.** Show that in a Boolean algebra, B:

- (i)  $a \vee (a \wedge b) = a \vee b$
- (ii)  $a \wedge (a \vee b) = a \wedge b$

**Q. 10.** Evaluate:  $\lim_{x \rightarrow 0} \frac{x \tan 4x}{1 - \cos 4x}$

**Q. 11.** Differentiate  $\sqrt{\cos x}$  w.r.t.  $x$  from first principles.

**Q. 12.** Differentiate  $\cot^{-1}\left(\frac{1-x}{1+x}\right)$  w.r.t.  $x$ .

**Q. 13.** Find the equations of the tangent and the normal to the curve

$x = 1 - \cos \theta, y = \theta - \sin \theta$  at  $\theta = \frac{\pi}{4}$ .

**Q. 14.** Evaluate:  $\int \frac{1}{3 + 2 \sin x + \cos x} dx$ .

**Q. 15.** Evaluate:  $\int_0^{\pi/2} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$ .

**Q. 16.** Using matrix method solve the following system of linear equations:

$x + y + z = 3$   
 $2x - y + z = 2$   
 $x - 2y + 3z = 2$

**Q. 17.** Show that a right circular cylinder which is open at the top, and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

**Q. 18.** Using integration, find the area of the circle  $x^2 + y^2 = 16$  which is exterior to the parabola  $y^2 = 6x$ .

Or

Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the line

$\frac{x}{a} + \frac{y}{b} = 1$ .

**SECTION - B**

**Q. 19.** If  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ , then show that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are orthogonal.

Or

Find x such that the four points  $A(3, 2, 1)$ ,  $B(4, x, 5)$ ,  $C(4, 2, -2)$  and  $D(6, 5, -1)$  are coplanar.

**Q. 21.** Find the Cartesian and vector equations of a line which passes through the point  $(1, 2, 3)$  and is parallel to the line.

**Q. 22.** Find the Cartesian equation of the sphere which has the points  $A(2, -3, 4)$  and  $B(-5, 6, -7)$  as the end points of one of its diameters, also find its centre and radius.

**Q. 23.** Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect. Find the point of intersection also.

**Q. 24.** Three forces  $\vec{P}$ ,  $\vec{Q}$  and  $\vec{R}$  acting on a point are in equilibrium. If the angle between  $\vec{P}$  and  $\vec{Q}$  is double the angle between  $\vec{R}$  and  $\vec{P}$ , prove that  $PQ = Q^2 - R^2$

**Q. 25.** Two unlike parallel forces  $\vec{P}$  and  $\vec{Q}$  ( $\vec{P} > \vec{Q}$ ) act at two points units apart. Show that if the direction of  $\vec{P}$  be reversed, the resultant will be displaced by a distance  $\frac{2PQ}{P^2 - Q^2} \cdot x$  units.

**Q. 26.** A particle starting with some initial velocity and moving with uniform acceleration acquires a velocity of 20 cm/sec after moving through 10 cm from a point P to Q, and a velocity of 30 cm/sec after further moving 20 cm from Q to R in the same direction. Find

- (i) its velocity at the point P.
- (ii) its acceleration.
- (iii) The time it will take and the distance

Or

From a point on the ground at a distance from the foot of a vertical wall, a ball is thrown at an angle of  $45^\circ$  which just clears the top of the wall and afterwards strikes the ground at a distance of  $y$  on the other side of the wall. Find the height of the wall.

### SECTION - C

**Q. 19.** Find the banker's discount and true discount on a bill of Rs. 22,800 due 4 months hence at 4% per annum.

**Q. 20.** A bill of exchange drawn on January 4, 2003 at 5 months date was discounted on March 26, 2003 at 3% per annum. If the banker's discount is Rs. 1207.20. Find the face value of the bill.

**Q. 21.** Three urns A, B and C contain 6 red and 4 white; 2 red and 6 white; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red. Find the probability that the ball was drawn from urn A.

**Q. 22.** The mean and variance of a binomial distribution are 4 and  $\frac{4}{3}$  respectively. Find  $P(X \geq 1)$ .

Or

If the sum of the mean and variance of a binomial distribution for 5 trials be 1.8, find the distribution.

**Q. 23.** A, B and C are partners in a business. A, being a working partner, receives 10% of the total profit as salary. The remaining profit is distributed among them in the ratio of 2 : 3 : 4. If A gets Rs. 3,00,000 in all, find the shares of B and C.

**Q. 24.** Find the present value of an annuity of Rs. 1,200 payable at the end of each 6 months for 3 years, when the interest is 8% per annum, compounded semi-annually. [Use  $(1.04)^6 = 1.2653$ ]

**Q. 25.** The total cost and demand function of an item are given by

$$C(x) = \frac{x^3}{3} = 7x^2 + 111x + 50$$

and  $p = 100 - x$  respectively.

Write the total revenue function and the profit function. Find the number of items when the profit will be maximum. Find the maximum profit also.

**Q. 26.** An oil company requires 13,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100,300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200,400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimise the cost of requirement?

Or

A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and only half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of A is Rs. 300 and

on one item of Rs. 160 How many items of each type should be produced to maximise the profit? Solve the problem graphically.