

## SECTION - A

**Q. 1.** If  $X = \begin{vmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{vmatrix}$

Or

$$X^n = \begin{vmatrix} \cos nA & \sin nA \\ -\sin nA & \cos nA \end{vmatrix}, n \in N.$$

Prove that

**Q. 2.** Using properties of determinants, Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \alpha & \gamma \\ \beta\gamma & \gamma\alpha & \alpha\beta \end{vmatrix} = (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha).$$

**Q. 3.** From a bag containing 20 tickets, numbered from 1 to 20 two tickets are drawn at random. Find the probability that

- (i) Both the tickets have prime numbers on them;
- (ii) On one there is prime number and on the other there is a multiple of 4.

**Q. 4.** Two dice are tossed once. Find the probability of getting an even number on the first dice or a total of 8.

Or

From a lot of 30 bulbs, which includes 6 defective bulbs, a sample of 3 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

**Q. 5.** Evaluate:  $\int \frac{\sin x}{\sin(x-a)} dx.$

**Q. 6.** Evaluate:  $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx.$

**Q. 7.** Solve the differentia equation:  $(y+x)\frac{dy}{dx} = y-x.$

**Q. 8.** Solve the differential equation:  $\frac{d^2y}{dx^2} = x^2 + \sin 3x$

Given that  $\frac{dy}{dx} = 1 = y$ , when  $x = 0$ .

**Q. 9.** For each  $x$  in a Boolean Algebra  $B$ , prove that  $\left. \begin{array}{l} x + y = 1 \\ x \cdot y = 0 \end{array} \right\} \Rightarrow y = x'$

**Q. 10.** Evaluate:  $\lim_{x \rightarrow \pi} \frac{\sin 3x - 3 \sin x}{(\pi - x)^3}$

Or

Evaluate:  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{x^2}$

**Q. 11.** Differentiate w.r.t.  $x$ :

$$\tan^{-1} \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}$$

**Q. 12.** Differentiate  $\sqrt{x}$  w.r.t.  $x$  from first principles.

**Q. 13.** The volume of a spherical balloon is increasing at the rate of  $25 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area at the instant when its radius is  $5 \text{ cm}$ .

**Q. 14.** Evaluate:  $\int \frac{2x}{(1+x^2)(3+x^2)} dx$ .

**Q. 15.** Prove that:  $\int_0^{\pi} \frac{x}{1 + \sin x} dx = \pi$ .

Or

Prove that:  $\int_0^{\pi/4} \sqrt{1 - \sin 2x} dx = \sqrt{2} - 1$ .

**Q. 16.** Using matrix method solve the following system of linear equations:

$$x + y - z = 1$$

$$3x + y - 2z = 3$$

$$x - y - z = -1$$

Or

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

If

Prove that:  $A^2 - 4A - 5I = O$ . Hence find  $A^{-1}$ .

**Q. 17.** Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius  $5\sqrt{3}$  cm is  $500\pi$  cm<sup>3</sup>.

**Q. 18.** Find the area bounded by the circle  $x^2 + y^2 = 16$  and the line  $y = x$  in the first quadrant.

### SECTION - B

**Q. 19.** If the vectors  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$  are coplanar, find the value of  $\lambda$ .

**Q. 20.** Prove that  $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

Or

The volume of the parallelepiped whose edges are  $-12\hat{j} + \lambda\hat{k}$ ,  $3\hat{j} - \hat{k}$  and  $2\hat{j} + \hat{j} - 15\hat{k}$  is 546 cubic units, find the value of  $\lambda$ .

**Q. 21.** The Cartesian equations of a line are  $3x + 1 = 6y - 2 = 1 - z$ . Find the fixed point through which it passes its direction ratios and also its vector equation.

**Q. 22.** Find the equation of the plane passing through the points (0, -1, 0), (1, 1, 1) and (3, 3, 0).

**Q. 23.** Prove that the plane  $x + 2y - z = 4$  intersects the sphere  $x^2 + y^2 + z^2 - x + z - 2 = 0$  in a circle of radius unity. Also the centre of the circle.

**Q. 24.** Three forces  $\vec{P}, \vec{Q}$  and  $\vec{R}$  act along O A, OB and OC, where O is the orthocenter of the  $\triangle ABC$ . If the forces be in equilibrium, prove that  $P : Q : R = a : b : c$ .

**Q. 25.** The resultant of two like parallel forces  $\vec{P}$  and  $\vec{Q}$  acting at A and B, 8 cm apart is 40 N. If the resultant passes through C, where AC=3 cm, find the magnitude of the forces.

**Q. 26.** A particle moving in a straight line with uniform acceleration describes successive equal distance in times  $t_1, t_2$  and  $t_3$

Prove that 
$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

Or

A stone is projected at an angle  $\alpha$  with the horizontal. Given its velocity when it attains half of the maximum height (it can attain) is  $\sqrt{2}$  times of the velocity at the maximum height, prove that  $\tan \alpha = \sqrt{2}$ .

### SECTION - C

**Q. 19.** The true discount and banker's gain on a certain bill of exchange due after a certain period of time are respectively Rs. 700 and Rs. 17.50. Find the face of the bill.

**Q. 20.** A bill of Rs. 1,000 drawn on May 7, 2003 for six months was discounted on August 29, 2003 for a cash payment of Rs. 988. Find the rate of interest charged by the bank.

**Q. 21.** A company has two plants to manufacture T. V. s. The plant manufactures 70% of the T.V.s. and the rest are manufactures by the other plant. 80% of the T.V.s. manufactured by the first plant are rated of standard quality, while that of the second plant only 70% are of standard quality. If a T.V. chosen at random is found to be of standard quality, find the probability that it was produced by the first plant.

**Q. 22.** A pair of dice is thrown 7 times. If getting the total 7 is considered a success find the probability of (i) no success, (ii) at least 6 succes.

Or

If the probability that an individual suffers from reaction by an injection is  $\frac{1}{1000}$ , find the probability that out of 5000 individuals given that injection (i) exactly 5 will suffer from reaction, (ii) no one will suffer reaction. Given that  $e^{-5} = 0.00672$ .

**Q. 23.** X and Y entered into a joint business with their capitals in the ratio 3 : 2. At the end of 3 months, X took out one-third of his capital, but after another 3 months Y put in a sum equal to what X had taken out. If at the end of the year y gets Rs. 11,000 more than what X got as profit find

- (i) the total profit in the business;
- (ii) the profit share of X in the business.

**Q. 24.** A television set is available for Rs. 20, 000 cash or Rs. 5,000 as cash down payment followed by 6 equal annual instalments, the first to be paid one year after the date of purchase. If the rate of interest under the instalment plan is 10% per annum, determine the amount of instalment. [Given that  $(1.1)^{-6} = 0.5644$ ]

**Q. 25.** If the cost function of an article manufactured by a company is given by

$$C(x) = 300x - 10x^2 + \frac{1}{3}x^3, \quad \text{find the output at which}$$

- (i) The marginal cost is minimum;
- (ii) The average cost is minimum.

**Q. 26.** A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost, satisfying the requirements.

Or

Solve the following linear programming problem graphically: